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Improved Procedure for Combining Day-of-Launch Atmospheric Flight Loads

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An improved procedure for combining launch-vehicle atmospheric flight load contributors is presented. This new procedure was applied to loads from a heavy-lift launch vehicle and was found to produce load-to-allowable value ratios with lower bias and less variance than a widely used procedure. It is expected that the new procedure will increase launch availability without reducing launch reliability.

Nomenclature

C_K	= coefficient of order K Hermite polynomial
f_X	= density function of a random variable x
H_K	= order K Hermite polynomial
L_{ALLOW}	= structural allowable load, lb or in.-lb
L_{AXIAL}	= axial load, lb or in.-lb
L_{EQ}	= equivalent axial load, lb or in.-lb
M_P	= pitch bending moment, lb or in.-lb
M_T	= combined yaw and pitch bending moment, lb or in.-lb
M_Y	= yaw bending moment, lb or in.-lb
m_3	= third central moment
$N(\mu, \sigma)$	= normal random variable with mean μ and standard deviation σ
P	= random variable representing M_P
Pr	= probability
$U(a, b)$	= uniform random number with minimum a and maximum b
X	= random variable representing M_X
Y	= random variable representing M_Y
μ_X	= mean of random variable X
σ_X	= standard deviation of random variable X

Introduction

MOST launch vehicles can only achieve the desired level of structural reliability by restricting the winds through which the vehicle is allowed to fly. This restriction is accomplished by first analytically flying the vehicle through wind profiles that are measured just prior to launch and calculating altitude histories of angles of attack, dynamic pressure, rigid-body acceleration, and engine gimbal angles, among others. These time histories are then used to establish static-aeroelastic and other day-of-launch calculated loads, which are then combined with the pre-day-of-launch calculated loads to obtain the total load.[1,2] Because many of the individual loads have random characteristics, a statistical load enclosure of the total load is computed. This total load enclosure is then compared to the vehicle allowable strength, and if it is exceeded, the vehicle is not launched. If sufficient time is available before the end of the launch window, the entire process is repeated for subsequent wind profiles. If there is not enough time, the launch attempt is aborted, and the vehicle is prepared for the next available launch window.

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For the launch vehicle core structure it is typical to express the comparison between the day-of-launch calculated load enclosures and the allowable values as capability ratios (CRs), i.e.,

$$\mathrm{CR} = \frac{\text{equivalent axial load}}{\text{structural allowable load}} = \frac{L_{\mathrm{EQ,MAX}}}{L_{\mathrm{ALLOW}}} \tag{1}$$

The structural allowable load L_{ALLOW} is determined prior to the day of launch, usually with structural testing and appropriate factors of safety. The equivalent axial load L_{EQ} is determined from an evaluation of the stress at a cross section of the launch vehicle. It is a function of the resultant bending moment M_T, obtained from the combination of the yaw and pitch bending moment components M_Y and M_P. It also depends on the axial load L_{AXIAL} and the radius to the point on the launch vehicle cylinder wall:

$$L_{\mathrm{EQ,MAX}} = L_{\mathrm{AXIAL,MAX}} + \frac{2M_T}{\text{radius}} \tag{2}$$

The combined pitch and yaw bending moments M_T are computed by combining a set of statistically varying loads from a variety of sources. To ensure, with a high probability, that the predicted equivalent axial load is not exceeded during atmospheric flight, a load enclosure of M_T is used in Eq. (2) to compute the CRs. The load enclosure is usually computed at a 99.7% enclosure, 90% confidence, level.[3]

The enclosure load is computed by combining loads obtained from analyses performed to simulate the various atmospheric flight load contributors. Static-aeroelastic (STEL) loads analyses are performed to establish the loads that are caused by that portion of the vehicle's angle of attack that varies relatively slowly with time. The STEL load is a function of the day-of-launch winds and the vehicle steering profile. Gust analyses are performed to establish the response of the vehicle, and its payload, to the turbulence that might be encountered on any given flight. Buffet analyses are performed to establish the loads caused by the dynamic response of the vehicle/payload system to shock waves, flow separation caused by changes in vehicle geometry, and the interaction between the two. In addition, other analyses are often performed to estimate loads from items such as vehicle load alleviation steering, autopilot noise, wind measurement error, changes in day-of-launch winds from the time they are measured to when the vehicle is to be launched, and vehicle dispersions from the nominal parameters used in the analyses.

The most accurate method of computing the enclosure load is by Monte Carlo simulation.[4] Because of time constraints, Monte Carlo simulation is currently not a realistic computational option during day-of-launch operations. An analysis and launch decision is required within a matter of a few minutes, and a Monte Carlo simulation requires considerably more time. To quickly calculate CRs on the day of launch, a computationally efficient analytical procedure is necessary.

A widely used analytical method for computing the enclosure load uses what are referred to as loads combination equations (LCEs).[1,2,5] Enclosure loads and the resulting CRs computed using the LCEs

Table 1 Predicted bending moments (million in.-lb)

			Yaw plane		Pitch plane	
Source	Computation	Distribution	Mean	Standard deviation	Mean	Standard deviation
STEL	DOL	——	97.53	0	635.61	0
Lack of persistence	DOL	Normal	86.11	141.87	68.95	109.37
Measurement error	DOL	Normal	0	82.59	0	91.08
Trajectory dispersion	Pre-DOL	Normal	0	100.18	0	96.28
Aerodynamic	Pre-DOL	Normal	0	3.16	0	19.91
Maneuvering	Pre-DOL	Normal	167.49	24.56	147.55	19.86
Wind data gap	Pre-DOL	Normal	0	0	0	0
			rss		*rss*	
Buffet	Pre-DOL	Rayleigh	119.72		141.22	
Autopilot	Pre-DOL	Rayleigh	0		0	
			Response to 30-ft/s gust		*Response to 30-ft/s gust*	
Gust	Pre-DOL	Gamma	1976.58		1909.37	

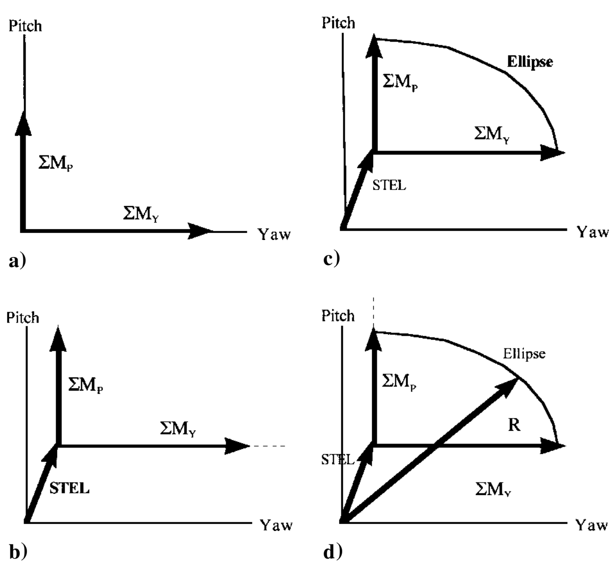

Fig. 1 LCE method for computing the limit load.

have been shown to be biased on the conservative side. This bias reduces launch availability. Reduction factors have been incorporated into the LCEs to reduce the conservative bias. However, this can also, on occasion, lead to underpredicting the CRs.

This paper presents the derivation of a new procedure, referred to as the bivariate integration load combination equation (BILCE), that has less bias and variance than other LCE procedures. Comparisons of the performance of the BILCE and LCE are made by using both to derive load CRs from several heavy-lift launch-vehicle load datasets and comparing these to load CRs computed using a Monte Carlo simulation.

Day-of-Launch Implementation of LCE

An example of an actual LCE input load dataset from the day of launch (DOL) of a heavy-lift launch vehicle is given in Table 1. Only the STEL, lack of wind persistence, and wind measurement error loads are established on the DOL because these depend on the DOL winds and steering profile. The remaining loads are computed prior to the DOL. Most of the loads are assumed to follow a normal distribution, except the buffet and autopilot loads, which are assumed to follow a Rayleigh distribution,[6] and the gust load, which is assumed to follow a gamma distribution.[7]

A pictorial of how the enclosure load is determined is given in Fig. 1, where ΣM_Y and ΣM_P are the yaw and pitch vector sums (summed means + rss of dispersed terms) and R is the 99.7% enclosure load. The steps to compute the enclosure load are the following:

1) Compute the mean and dispersed term of each load in the yaw and pitch directions. The dispersed term is the load's 99.7% minus its mean.

2) Reduce the means and dispersed terms of the gust, buffet, autopilot, trajectory, and maneuvering loads by 10%, as a typical value. This is the reduction factor obtained from a Monte Carlo simulation.[1]

3) Add the sum of the means (excluding STEL) with the rss of the dispersed terms in the yaw and pitch planes (Fig. 1a).

4) Translate the yaw and pitch vector sums to the STEL's yaw and pitch vector load (Fig. 1b).

5) Construct an ellipse such that major and minor axis vertices are the translated yaw and pitch vector sums (Fig. 1c).

6) The enclosure load is the magnitude of the point on the ellipse that is the maximum distance from the origin (Fig. 1d).

Further details of the methodology used in this LCE can be found in Ref. 1. The LCE has sufficed for many years and many launches by various heavy- and medium-lift launch vehicles. Although it has served its purpose, the approach has a certain inherent bias and variability that are because of the following reasons:

1) The mean sum added to the rss of the dispersed terms (steps 3 and 4) is not a 99.7% enclosure for the sum of nonnormal loads. An example to illustrate this is the sum of two independent $U(0, 1)$ random variables. The 99.7% of the sum is 1.92, whereas the mean sum added to the rss of the dispersed terms is 1.70.

2) The magnitude of the point on the ellipse that is the maximum distance from the origin (steps 5 and 6) is not the 99.7% bound of the load magnitude, even if the vertices of the ellipse are the true 99.7%. An example here is the rss of two independent $N(0, 1)$ random variables where the center of the ellipse is the origin. The 99.7% of the rss is 3.41, whereas the magnitude of the line from the origin to the point on the ellipse furthest from the origin is 3.89. Even with these approximations, Monte Carlo simulations have shown that the LCE generally performs as intended. However, it is now possible to combine the various load contributors in a more rigorous manner. It is the purpose of this paper to introduce such a procedure.

DOL Implementation of the BILCE

The enclosure load of the bending moment is the value R such that

$$Pr\left(\sqrt{Y^2 + P^2} \le R\right) = 0.997 \tag{3}$$

Given that Y and P have density functions f_Y and f_P, Eq. (3) can be rewritten as

$$\iint_{\sqrt{u^2+v^2} \le R} f_Y(u) f_P(v)\, \mathrm{d}u\, \mathrm{d}v = 0.997 \tag{4}$$

Because of the nonnormal loads (buffet, autopilot, and gust) and the numeric difficulties in convoluting their density functions with the other bending moment densities, computing f_Y and f_P and consequently the exact value of R in a timely manner during DOL operations is time consuming.

Because Y is the sum of several statistically independent loads, from the central limit theorem, its density function f_Y is approximately normal with mean μ_Y, the sum of the individual load's means, and standard deviation σ_Y, the rss of the individual load's standard deviations. This approximation is sufficient as long as the dominating loads are approximately normal. Unfortunately, two significant load contributors, buffet and gust, have Rayleigh and gamma distributions, respectively, which are both highly skewed to the right. The central limit theorem approximation can be improved[8] by modifying the normal density approximation so that its higher central moments are equal to the higher central moments of Y. The modified density approximation can be written as

$$f_Y(u) \approx \frac{1}{\sigma_Y\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{u-\mu_Y}{\sigma_Y}\right)^2\right]\left[1+\sum_{k=1}^{\infty} C_k H_k\left(\frac{u-\mu_Y}{\sigma_Y}\right)\right] \tag{5}$$

where $H_K(x)$ is a Hermite polynomial of order k and C_K are coefficients determined by equating the central moments of Y with the density approximation. Equating only the first three moments, we find

$$C_1 = C_2 = 0, \qquad C_3 = m_3 / 6\sigma_Y^3 \tag{6}$$

i.e., $[E(Y-\mu_Y)^3]$. Because

$$H_3(x) = x^3 - 3x \tag{7}$$

Eq. (5) can be written as

$$f_Y(u) \approx \frac{1}{\sigma_Y\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{u-\mu_Y}{\sigma_Y}\right)^2\right] \times\left\{1+\frac{m_3}{6\sigma_Y^3}\left(\frac{(u-\mu_Y)^3}{\sigma_Y^3}-\frac{3(u-\mu_Y)}{\sigma_Y}\right)\right\} \tag{8}$$

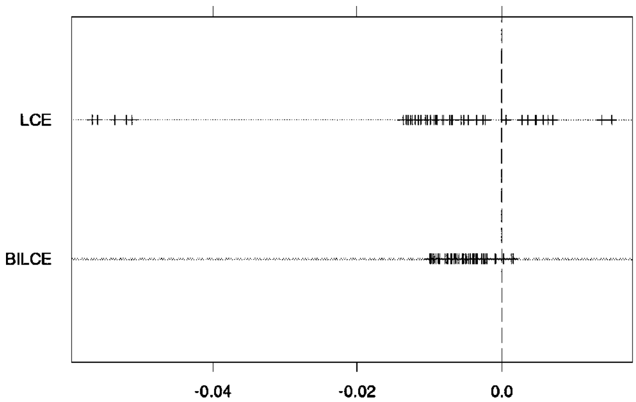

Fig. 2 CR difference for a heavy-lift launch vehicle at Mach 1.58.

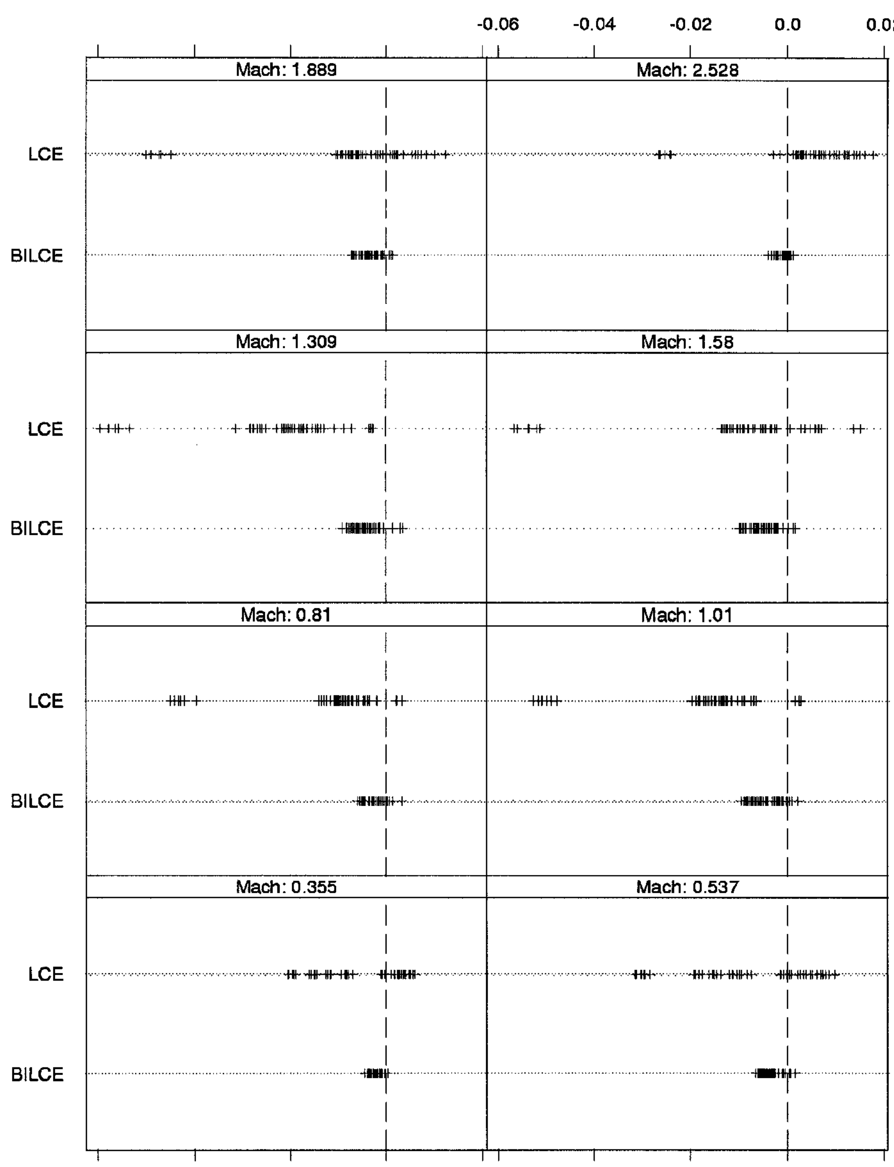

Fig. 3 CR difference for a heavy-lift vehicle at given Mach bands.

The approximate density function of P is determined in an analogous manner. Using Eqs. (4) and (8), the steps to compute the BILCE enclosure load are the following:

1) Compute the mean, standard deviation, and third moment (skewness) of each load in the yaw and pitch direction. These statistics can be computed analytically or by Monte Carlo techniques for the pre-DOL loads.

2) Sum the means, variances, and third moments to determine μ, σ, and m_3.

3) Approximate the density functions f_Y and f_P of the bending moment sums Y and P, using Eq. (8).

4) The enclosure load is the value R in Eq. (4) found by using Romberg numerical integration[2] and the approximate density functions f_Y and f_P.

Though higher moments can be used to approximate f_Y and f_P for the launch-vehicle flights examined, the first three moments were adequate to account for the skewness as a result of the buffet and gust loads.

Accuracy of the BILCE

The accuracies of the LCE and the BILCE are assessed by comparing their CRs with CRs computed using a Monte Carlo simulation. Details of the Monte Carlo simulation are given in Refs. 3, 4, and 9. Comparisons are made at several launch-vehicle stations and flight Mach numbers from nine heavy-lift launch vehicle flights.

A comparison of the CRs for one of the flights at Mach 1.58 is shown in Fig. 2. The marks that overlay the top horizontal line are CR differences (Monte Carlo CR–LCE CR) derived at 48 launch-vehicle stations. The bottom marks are CR differences at the same stations using the BILCE.

The reduction in variation, using the BILCE compared to the LCE, demonstrates the improved accuracy of the method. For the BILCE the lack of points far to the left of zero indicates fewer overpredictions of the enclosure load compared to the LCE, providing the potential for making fewer unnecessary "No-Go" launch decisions by using the BILCE. The lack of points far to the right indicates the potential to also make fewer "Go" launch decisions with lower than the desired statistical enclosure.

The CR comparisons for the same flight at the other Mach numbers studied are given in Fig. 3. Results are similar to the Mach number 1.58 values; CR accuracy is improved using the BILCE. CR comparisons for the remaining eight heavy-lift launch-vehicle flights show similar results.

Conclusions

A refined procedure for combining atmospheric flight loads to compute load-to-allowable ratios has been presented. This procedure produces CRs with less bias and variation than a previously used procedure. As a result, DOL implementation of the proposed procedure should increase launch availability without reducing launch reliability.

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